Short Tricks - JEE-Main

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1. Progression

Lecture - 3

Q.15 If x, y, z are in A.P. and x, y, t are in G.P. then x, x – y, t – z are in -

(A) G.P. (B) A.P. (D) H.P. (D) A.P. and G.P. both

Sol. [A]

Proper Method	**Short Trick**
x, y, z are in A.P. $\Rightarrow 2y = x + z$ Or $2xy = x^2 + xz$ (multiplying with x) $\Rightarrow x^2 - 2xy = -xz$ ...(1) x, y, t are in G.P. $\Rightarrow y^2 = xt$ or $(x^2 - 2xy + y^2) = -xz + xt$ or $(x - y)^2 = x(t - z)$ x, x – y, t – z are in G.P.	Let x = 1, y = 2, z = 3 and t = 4 $x, x - y, t - z \equiv 1, -1, 1$ are in G.P.

Q.16 If a, b, c are in H.P. then $\frac{a}{b+c}$, $\frac{b}{c+a}$, $\frac{c}{a+b}$ will be in-

(A) A.P. (B) G.P. (C) H.P. (D) None of these

Sol. [C]

Proper Method

a, b, c are in HP

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b-c}{c}$ are in AP

$\Rightarrow 1+\frac{b+c}{a}, 1+\frac{c+a}{b}, 1+\frac{a+b}{c}$ are in AP

$\Rightarrow \frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in AP

$\Rightarrow \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in HP.

Short Trick

Let a = 2, b = 3, c = 6

then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$

$\equiv \frac{2}{9}, \frac{3}{8}, \frac{6}{5}$

which are in H.P

Q.17 If $a_1, a_2, \ldots\ldots a_n$ are in H.P., then $\frac{a_1}{a_2 + a_3 + \ldots\ldots + a_n}, \frac{a_2}{a_1 + a_3 + \ldots\ldots + a_n}, \ldots\ldots\ldots, \frac{a_n}{a_1 + a_2 + \ldots\ldots + a_{n-1}}$ are in

(1) A.P. (2) G.P. (3) H.P. (4) None of these

Sol. [C]

Proper Method	Short Trick
$a_1, a_2, \ldots a_n$ are in H.P. $\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n}$ are in A.P. $\Rightarrow \frac{a_1+a_2+a_3+\ldots+a_n}{a_1}, \frac{a_1+a_2+a_3+\ldots+a_n}{a_2}, \ldots,$ $\Rightarrow 1+\frac{a_2+a_3+\ldots+a_n}{a_1}, 1+\frac{a_1+a_3+\ldots+a_n}{a_2}, \ldots,$ $\frac{a_1+a_2+\ldots+a_{n-1}}{a_n}$ are in A.P. $\Rightarrow \frac{a_2+a_3+\ldots+a_n}{a_1}, \frac{a_1+a_3+\ldots+a_n}{a_2}, \ldots,$ $\frac{a_1+a_2+\ldots+a_{n-1}}{a_n}$ are in A.P. $\Rightarrow \frac{a_1}{a_2+a_3+\ldots+a_n}, \frac{a_2}{a_1+a_3+\ldots+a_n}, \ldots\ldots,$ $\frac{a_n}{a_1+a_2+\ldots+a_{n-1}}$ are in H.P.	**Short Trick** Let $a_1 = 2$, $a_2 = 3$, $a_3 = 6$ $\left.\begin{array}{l}\frac{a_1}{a_2+a_3} = \frac{2}{9} \\ \frac{a_2}{a_1+a_3} = \frac{3}{8} \\ \frac{a_3}{a_1+a_2} = \frac{6}{5}\end{array}\right\}$ are in H.P.

Q.18 If the sum of m terms of an A.P. is the same as the sum of its n terms, then the sum of its (m + n) terms is -

(1) mn (2) – mn (3) 1/mn (4) 0

Sol. [D]

Proper Method	Short Trick
Let a be the first term and d be the common difference the given AP then $S_m = S_n \Rightarrow \frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$ $\Rightarrow 2a(m-n) + \{m(m-1) - n(n-1)\}d = 0$ $\Rightarrow 2a(m-n) + \{(m^2 - n^2) - (m-n)\}d = 0$ $\Rightarrow (m-n)[2a + (m+n-1)d] = 0$ $\Rightarrow 2a + (m+n-1)d = 0$ Now $S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d] = \frac{m+n}{2} \times 0 = 0$	Let m = 2 and n = 3 $-2, -1, 0, 1, 2 \ldots$ sum of (m + n) terms = 0

Q.19 If x be the AM and y, z be two GM's between two positive numbers, then $\frac{y^3 + z^3}{xyz}$ is equal to-

(A) 1 (B) 2 (C) 3 (D) 4

Sol. **[B]**

Proper Method	Short Trick
Let numbers are a & b $\therefore x = \frac{a+b}{2}$ & $\because$ a, y, z, b are in GP $\therefore y^2 = az$ & $z^2 = by$ Now, $\frac{y^3+z^3}{xyz} = \frac{1}{x}\left(\frac{y^2}{z}+\frac{z^2}{y}\right) = \frac{1}{x}(a+b) = 2$	Let the G.P is 1, 2, 4, 8 then $y = 2$, $z = 4$ and $x = \frac{9}{2}$ Now $\frac{y^3+z^3}{xyz}$ $= \frac{8+64}{(2)\left(\frac{9}{2}\right)(4)} = 2$

Q.20 Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are-

(A) Not in A.P./G.P./H.P. (B) in A.P. (C) in G.P. (D) in H.P.

Sol. **[D]**

Proper Method	**Short Trick**
a, b, c, d : AP Dividing by abcd, we get $\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}$: AP $\therefore$ bcd, acd, abd, abc : HP	Let the A.P. is 1, 2, 3, 4, Now abc = 6, abd = 8, acd = 12, bcd = 24 are in H.P.

Q.21 Let the sequence $a_1, a_2, a_3, \ldots\ldots, a_n$ form an A.P., then $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots\ldots + a_{2n-1}^2 - a_{2n}^2$ is equal to -

(A) $\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$ (B) $\frac{2n}{n-1}(a_{2n}^2 - a_1^2)$ (C) $\frac{n}{n+1}(a_1^2 + a_{2n}^2)$ (D) None of these

Sol. [A]

Proper Method

Given $a_1, a_2, \ldots\ldots, a_{2n}$: AP

$\therefore\ a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots.. + a_{2n-1}^2 - a_{2n}^2$

$= (a_1 - a_2)(a_1 + a_2) + (a_3 - a_4)(a_3 + a_4) + \ldots\ldots.$

$+ (a_{2n-1} + a_{2n})(a_{2n-1} - a_{2n})$

But $a_1 - a_2 = a_3 - a_4 \ldots\ldots = a_{2n-1} - a_{2n} = -d$

$= -d[a_1 + a_2 + a_3 + a_4 \ldots\ldots\ldots.. + a_{2n}]$

$= -d \cdot \frac{2n}{2}[a_1 + a_{2n}]$

$= -nd(a_1 + a_{2n})$

But $a_{2n} = a_1 + (2n-1)d \Rightarrow d = \frac{a_{2n} - a_1}{2n-1}$

$= \frac{-n(a_{2n} - a_1)}{2n-1} \cdot (a_{2n} + a_1)$

$= \frac{n}{2n-1}(a_1^2 - a_{2n-1}^2)$

Short Trick

Let $n = 2$

$a_1 = 1,\ a_2 = 2,\ a_3 = 3,\ a_4 = 4$

$a_1^2 - a_2^2 + a_3^2 - a_4^2$

$= 1 - 4 + 9 - 16 = -10$

by option (A)

$\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$

$= \frac{2}{3}(1 - 16) = -10$